Converting Non-linear Models into Linear Ones

Absolute Value	
Domain and range	$x, l, u \in R$
Constant	
Upper bound	$u \leq \max(x, -x)$
Lower bound	$l \geq x$ $l \geq -x$
Usage	$\text{abs}(x) \in [l, u]$ $\min abs(x) \Rightarrow \min l$ $\max abs(x) \Rightarrow \max u \Rightarrow \max \max(x, -x) \Rightarrow \min \min(x, -x)$
Reference	https://optimization.mccormick.northwestern.edu/index.php/Optimization_with_absolute_values

Value of Minimal Element	
Domain and range	$X = \{x_1, x_2, \dots, x_{\|X\|}\}$ $x_i \in [a, b], l, u \in R, y_i \in \{0,1\}$
Constant	$a, b \in R$ $M > \max(\|a\|, \|b\|, b - a)$
Upper bound	$u \leq x_i, \quad \forall i \in [1, \|X\|]$
Lower bound	$l \geq x_i - M \cdot y_i, \quad \forall i \in [1, \|X\|]$ $\sum_i y_i = \|X\| - 1$
Usage	$\min(X) \in [l, u]$ $\min \min(X) \Rightarrow \min l$ $\max \min(X) \Rightarrow \max u$
Reference	http://math.stackexchange.com/questions/1858740/how-to-covert-min-min-problem-to-linear-programming-problem/1862709#1862709

Value of Maximal Element	
Domain and range	$X = \{x_1, x_2, \dots, x_{\|X\|}\}$ $x_i \in [a, b], l, u \in R, y_i \in \{0,1\}$
Constant	$a, b \in R$ $M > \max(\|a\|, \|b\|, b - a)$
Upper bound	$u \leq x_i + M \cdot y_i, \quad \forall i \in [1, \|X\|]$ $\sum_i y_i = \|X\| - 1$
Lower bound	$l \geq x_i, \quad \forall i \in [1, \|X\|]$
Usage	$\max(X) \in [l, u]$ $\max \max(X) \Rightarrow \max u$ $\min \max(X) \Rightarrow \min l$
Reference	http://math.stackexchange.com/questions/1858740/how-to-covert-

	min-min-problem-to-linear-programming-problem/1862709#1862709

Cardinality (Test Zero or Non-Zero)	
Domain and range	$X = \{x_1, x_2, \ldots, x_{\lvert X \rvert}\}$ $x_i \in [a, b], y_i \in \{0,1\}$
Constant	$a, b \in R$
Upper bound	$x_i \leq b \cdot y_i$
Lower bound	$x_i \geq a \cdot y_i$
Usage	$\mathrm{card}(X) = \sum_{\mathrm{i}} y_i$
Reference	

Boolean Logic	
Domain and range	$x, y \in \{0,1\}$
Constant	
Upper bound	
Lower bound	
Usage	$y = x_1 \wedge x_2 \wedge \ldots \wedge x_n \Rightarrow 0 \leq \sum_{i=0}^{n} x_i - n \cdot y \leq n - 1$ $y = x_1 \vee x_2 \vee \ldots \vee x_n \Rightarrow 0 \leq n \cdot y - \sum_{i=0}^{n} x_i \leq n - 1$
Reference	https://cs.stackexchange.com/questions/12102/express-boolean-logic-operations-in-zero-one-integer-linear-programming-ilp

"If-then" Logic (Implication)	
Domain and range	$x, z \in [a, b], y \in \{0,1\}$
Constant	$a, b \in R$ $M > \max(\lvert a \rvert, \lvert b \rvert, b - a)$
Upper bound	
Lower bound	
Usage	$z \leq x \cdot y \Rightarrow \begin{cases} z \leq x + M \cdot (1 - y) \\ z \leq M \cdot y \end{cases}$ $z \geq x \cdot y \Rightarrow \begin{cases} z \geq x - M \cdot (1 - y) \\ z \geq -M \cdot y \end{cases}$
Reference	http://math.stackexchange.com/questions/112159/what-are-the-algorithms-for-integer-programming-in-which-constraints-are-depende/112927#112927

Semi-continuous/integer Variable	
Domain and range	$x \in \{0\} \cup [a, b], y \in \{0,1\}$
Constant	$a, b \in R$

Upper bound	
Lower bound	
Usage	$x = 0 \text{ or } a \le x \le b \Rightarrow \begin{cases} x \ge a \cdot y \\ x \le b \cdot y \end{cases}$
Reference	https://math.stackexchange.com/questions/849319/forbidden-range-for-a-linear-programming-variable

Semi-binding Variable	
Domain and range	$x \in \{0, t\}, y \in \{0,1\}, t \in [a, b]$
Constant	$a, b \in R$ $M > \max(\lvert a \rvert, \lvert b \rvert, b - a)$
Upper bound	
Lower bound	
Usage	$x = 0 \text{ or } t \le x \le t \Rightarrow \begin{cases} -M \cdot (1 - y) \le x \le M \cdot (1 - y) \\ t - M \cdot y \le x \le t + M \cdot y \end{cases}$
Reference	

Linear-fractional Objective	
Domain and range	
Constant	
Upper bound	
Lower bound	
Usage	
Reference	https://en.wikipedia.org/wiki/Linear-fractional_programming Stancu-Minasian, I. M. *Fractional programming: theory, methods and applications*. Vol. 409. Springer Science & Business Media, 2012. Bajalinov, Erik B. *Linear-Fractional Programming Theory, Methods, Applications and Software*. Vol. 84. Springer Science & Business Media, 2013.